

18. KUO SHOU-CHING (1231-1316)

Kuo Shou-ching, T. Jo-ssu, was born in 1231 at Hsing-t'ai, in the Shun-te administration (Hopei), during the reign of the Chin emperor Ai-tsung (r. 1224-34). The names of his parents can no longer be traced, but it is known that his grandfather, Kuo Jung, was well-educated in the classics and mathematics, and had an interest in water works. Kuo Yung was a friend of his townsman, Liu Ping-chung (*q.v.*), later the Buddho-Taoist confidant of Qubilai Qa'an, who possessed a good knowledge of astronomy, mathematics, and geography.

As a child Kuo Shou-ching is said to have already shown traits setting him apart from other children, inasmuch as he took no pleasure in games and play. At the age of fourteen or fifteen, he came across a diagram of the 'Lotus Clepsydra' (*lien-hua lou*) and was able to understand at first sight the principle of this time-keeping device. (The 'Lotus Clepsydra' was so called because the top of the recipient vessel was shaped like a lotus flower.) The instrument had been devised by the Northern Sung scientist Yen Su and officially adopted in 1036. In 1246, Liu Ping-chung returned to Hsing-t'ai to observe the mourning period for his father and stayed there for a few years. During that time, Liu and a group of scholars, including Chang Wen-ch'ien (*q.v.*), Chang I (d. 1282), Wang Hsün (1235-81) and others were in the habit of meeting regularly for scholarly discussions at Tzu-chin Mountain, west of Hsing-chou (Shun-te). All were versed in classical scholarship, and especially in astronomy and mathematics. Kuo Yung seized this opportunity to make his grandson study under Liu Ping-chung, most likely sometime between 1246 and 1249. In this way, Kuo Shou-ching became known to the other scholars, especially to Chang Wen-ch'ien who later recommended him to Qubilai.

In late 1251, as a lad of twenty, Kuo Shou-ching got his first chance to demonstrate his scientific talent. The countryside north of Hsing-chou was encompassed by three converging rivers, the Liao-shui, the Ta-huo ch'üan, and the Kang-shui, all flowing westward. When the Mongols overran China, the lower course of the Liao-shui became obstructed, which caused the waters to bank up in a vast muddy marsh. There were stone bridges on all three rivers, but that over the Ta-huo ch'üan was washed away and no trace of its foundations could be found. It was replaced in due course by a temporary wooden structure so flimsy that it kept collapsing and had to be rebuilt repeatedly over the years, causing the local people a great deal of inconvenience. In autumn 1251, Chang Keng and Liu Su (1188-1263) were appointed pacification commissioners of the region by Möngke. Anxious to

resolve the problems of the bridge, they took counsel with the local elders and with such as knew the river, and inspected the site personally. Informed of Kuo Shou-ching's talents, they had him produce plans and cost estimates, and put him in charge of the restoration project. Kuo surveyed the area and located the site of the old bridge. He cleared the mud that covered its footings and had the structure rebuilt, thus ensuring easy passage once again. The project required only four hundred labourers and was completed in forty days. This feat was duly recorded by the great former Chin literatus Yüan Hao-wen (1190-1257) in a commemorative document written for this occasion.

In June 1260, after Qubilai's enthronement, Chang Wen-ch'ien was appointed assistant of the Left in the Secretarial Council, and shortly thereafter was despatched to Ta-ming, Chang-te, and adjacent districts, to overhaul the local administration. He took Kuo Shou-ching with him, which gave the young engineer an opportunity thoroughly to survey the irrigation systems of the region. At the same time, Kuo had undertaken to improve the instruments of astronomy. He cast a bronze clepsydra (*t'ung-hu*) for the local observatory; constructed a bamboo *hsüan-chi* instrument after a diagram from the *Shu-ching* (*Book of History*); and also a tower for observing the twenty-eight lunar mansions (*hsiu*), as well as the larger stars and the constellations. (Whereas Michel interprets the *hsüan-chi* as a template, Kuo's apparatus seems more likely to have been an armillary sphere.) The clepsydra, known as the Pao-shan clepsydra, was completed in March 1262 and was subsequently moved to the capital city Yen-ching (later Ta-tu), i.e. the present-day Peking.

Greatly impressed, Chang Wen-ch'ien recommended Kuo Shou-ching to Qubilai in 1262, praising his expertise in water works and his extraordinary intelligence. In August, the qa'an granted Kuo an audience at Shang-tu. On that occasion, Kuo presented six different schemes for improving irrigation and waterways in the region south of Yen-ching and north of the Yellow River. They were:

1. The water flowing from Yü-ch'üan Mountain (the famous 'Jade Fountain') north-west of the capital could be diverted into the disused canal constructed by the Chin, which had become unserviceable by Kuo's time. This would reactivate the canal connecting the capital and T'ung-chou (in Hopei) and save 60,000 *min* in road transport expenses annually. Furthermore, an extension of the canal from the mouth of the Lin-yü River south of T'ung-chou, to Yang-ts'un (north-west of Tientsin), would present three advantages: it would avoid the shallow and muddy stretches of the river; it would offer shelter from wind and waves; and it would eliminate the need for transshipping cargo.

2. The water from the Ta-huo ch'üan to the north of Shun-te could be diverted to run into the walled city so that it would flow eastwards in three tributaries. The water could be used to irrigate the farms to the east of the city.
3. The Li River, which flowed northwards from south of Shun-te to Jen-ch'eng (also in Hopei), was blocked by mud at that time and had inundated more than 1,300 *ch'ing* of farm land. With proper drainage the farms could become arable once again and the Li River would flow towards Hsiao-wang ts'un where it would join the Hu-t'o River and enter the Yü River (i.e. the Wei River) and would be navigable all the way.
4. At the junction of the Fu and Chang rivers north-east of Tz'u-chou (Hopei), a canal could be dug to lead the waters northwards to Han-tan and Yung-nien, and via Chi-tse (north-west of Ch'ü-chou, Hopei) into the Li River, providing irrigation for over 3,000 *ch'ing* of farm land.
5. Although the course of the Ch'in River between Huai-chou and Meng-chou (Honan) provided some irrigation facilities, the channel was overloaded and banks had been broken by excess water. Irrigation for over 2,000 *ch'ing* could be provided if the latter were diverted to collect the overflow from the Tan before being led into the Yü River north of Wu-ch'ih.
6. West of Meng-chou, a waterway could be dug on the northern shore of the Yellow River, running north then east between the new and old sites of Meng-chou and then along the course of the Yellow River to Wen county before it rejoined the Yellow River further south. This would irrigate more than 2,000 *ch'ing* of farm land.

Kuo's presentation greatly impressed Qubilai, who immediately appointed him superintendent of waterways for the various districts. The first of Kuo's suggestions was actually implemented. The following year Kuo was rewarded with promotion to the rank of vice-commissioner of waterways, and was granted the silver tablet (*yin-fu*).

Early in 1264, in his capacity as assistant of the Left in the Secretarial Council, Chang Wen-ch'ien was appointed assistant of the Left in the Regional Secretarial Councils of Hsi-Hsia, Chung-hsing and neighbouring districts, comprising the territories of the defunct Tangut state in Kansu and Shensi. Once again, he took Kuo and a Mongol official, Ai Dö'en, with him to restore the irrigation systems along the Yellow River in Hsi-Hsia that had become blocked or damaged during the years of war. Those systems, the most extensive of which was the T'ang-lai ch'ü, (ca. 220 km in length) and the second largest, the Han-yen ch'ü (ca. 140 km), included more than

ten main channels flowing from the Yellow River into some sixty-eight branch canals. The area irrigated by this network amounted to some half a million hectares. After Kuo had completed his mission, the Tangut people built a temple on the bank of a canal to make offerings to him even though he was still alive.

The following year Kuo Shou-ching was appointed vice-director of waterways on his return to the capital. On his way back he had sailed four days and nights on the Yellow River to Tung-sheng (in Inner Mongolia) and discovered that this particular stretch of the river was navigable for large cargo boats. He also went past the Cha-po-wu-lang Sea (Wu-liang-su, in Inner Mongolia) and observed many derelict canals that might be restored. These findings were duly reported to the Mongol court. Kuo may thus have been the first to survey the sources of the Yellow River. Similarly, Kuo drew attention to another old canal constructed under the Chin between Ma-ku ts'un (north-west of Shih-ching Mountain, and some 20 km west of Yen-ching) and the Lu-kou River (i.e. the Yung-ting River). Its waters came out of the mountain range to the west of the capital, at a place known as Chin-k'ou. A vast area of farm land between Chin-k'ou and the capital used to be irrigated from this canal, but the outflow at Chin-k'ou had been blocked during the war between the Mongols and the Jurchens, after which the responsible government official was loath to reopen the outlet for fear of flooding downstream. Kuo suggested that the old canal be widened and deepened, so that it could be used both for navigation and for irrigation while safely draining away any excess water. This suggestion was adopted.

In 1271 Kuo Shou-ching was made director of waterways. Later that year Qubilai adopted the dynastic title of 'Great Yüan' (Ta-Yüan), and renamed the capital Ta-tu shortly thereafter. The emperor then launched his troops against the Southern Sung; the Mongol forces, under the command of Bayan (*q.v.*), met with great success in several campaigns. In 1275 Bayan commanded Kuo to survey the plain at the confluence of the Yellow and Huai rivers with a view to building ports and waterways for the supply of the southern front with men and material. Kuo produced a map indicating how Chi-chou, Ta-ming, and Tung-p'ing could be linked by waterways with the Ssu, Wen and Yü rivers, which he presented to the emperor. He also compared the elevations above sea-level of the capital, P'ei county and Lü-liang (Honan). In the following year, the Directorate of Waterways was reorganised and absorbed into the Ministry of Works; Kuo thus lost his post in the former department and became a first secretary in the ministry.

At that time, the calendar in official use was the revised *Ta-ming li* adopted by the Chin court in 1182. (The original *Ta-ming li* was compiled in the Liu-Sung period, 420-79.) But, after more than ninety years, its accuracy had deteriorated; indeed, the solar cycle had fallen out of step with the lunar cycle by as much as two days. Yeh-lü

Ch'u-ts'ai (*q.v.*) had earlier suggested adopting the *Keng-wu yüan-li* calendar which he had compiled to replace the *Ta-ming li*, and the Persian astronomer Jamāl al-Dīn had also presented a *Wan-nien li* calendar based on the Muslim system (*Hui-hui li-fa*) to Qubilai in 1267. Neither system was accepted. A few years before his death in 1274, Liu Ping-chung had memorialised the emperor on the need for reforming the calendar and proposed to undertake the task himself, but he died shortly thereafter. In 1276, when the capital of Southern Sung, Lin-an (Hang-chou), had fallen to the Mongol army, Qubilai considered that the time had come to introduce a new calendar. In July of that year, he commissioned a special bureau to that effect, with Wang Hsün, then tutor to the heir apparent Jīngim and an expert mathematician, and also Chang I, in charge of the task. The calendrical bureau passed subsequently under the supervision of Chang Wen-ch'ien, then a senior academician of the Chao-wen Academy. In addition to Wang Hsün and Chang I, Chang invited Kuo Shou-ching and Hsü Heng (*q.v.*), the retired rector of the National College and an expert in calendar science, to lead a group of astronomers in making and compiling observations of the sun at an array of latitudes.

According to Kuo, the accuracy of the calendar depended on the accuracy of the observations, which in turn depended on the precision of the astronomical instruments employed. He pointed out that the armillary sphere used at that time at the capital Ta-tu had been made by the Sung at their own capital of Pien (K'ai-feng) and that there was a difference of about 4 degrees between the two sites; moreover, the gnomon erected by the Sung had become tilted with the passage of time. Kuo then set about to construct seventeen different astronomical instruments, the first thirteen for use in the observatory and the other four for field work. These instruments were:

1. *Chien i* ('Simplified Instrument'). This is the equatorial torquetum. A replica, made by the Ming astronomer Huang-fu Chung-ho in 1437, is displayed at the Tzu-chin Mountain Observatory, Nanking. It looks like a medieval Arabic and European torquetum without the ecliptic components – which makes it equatorial and justifies the name 'simplified.' Indeed, Kuo appears to have adapted the conventional torquetum which read ecliptic coordinates to one which read equatorial coordinates directly. The original instrument was examined but described inaccurately by Matteo Ricci in the early seventeenth century.
2. *Kao-piao* ('High Gnomons'). Giant gnomons, some 12 m high. There is an example of this instrument at Yang-ch'eng which is described by Tung Tso-pin and his collaborators.

3. *Hou-chi i* ('Pole-Observing Instrument'). Probably a pole-sighting tube provided at its upper end with a 'pole-determining circle.' The tube might have been included in the equatorial torquetum in (1) above.
4. *Hun-t'ien hsiang* ('Celestial Globe'). This was a large globe and, according to Ricci, three men with outstretched arms could hardly have embraced it. It was provided with meridians and declination circles parallel to the equator. Any markings of stars that it probably carried originally would have been erased by exposure to the elements in Ricci's time.
5. *Ling-lung i* ('Ingenious Armillary Sphere'). This instrument was examined and described by Ricci, who said:

The second instrument was a great (armillary) sphere not less in diameter than that measure of the outstretched arms which is commonly called a geometric pace. It had a horizon (-circle) and poles; instead of (solid) circles it was provided with certain double hoops (armillae), the void space between the pair serving the same purpose as the circles of our spheres. All these were divided into 365 degrees and some odd minutes. There was no globe representing the earth in the centre, but there was a certain tube, bored like a gun-barrel, which could readily be turned about and placed at any degree or altitude so as to observe any particular star, just as we do with our vane sights – not at all a despicable device. (From Needham, III:368.)

This tube, which appeared so strange to Ricci, was of course the sighting-tube used in Chinese astronomy. A replica of this equatorial armillary sphere made in 1437 by Huang-fu Chung-ho can be seen in the grounds of the Tzu-chin Mountain Observatory. However, Po Shu-ren (Po SR82) suggests that what Ricci saw and described was not the *ling-lung i*, but only the *hun-i* (armillary sphere), and that (5) was probably a large copper sphere with perforations on its surface to mark the positions of the stars.

6. *Yang-i* ('Upward-Looking Instrument'). A hemispherical sundial.
7. *Li-yün i* ('Vertical Revolving Circle'). The vertical circle in the equatorial torquetum in (1) above. Together with the azimuthal circle in (14) below, this would be equivalent of modern altazimuths and theodolites.
8. *Cheng-li i* ('Verification Instrument'). An instrument used to determine the exact positions of the sun and the moon near the ecliptic and above the horizon. It might have been an ecliptically mounted sighting-tube, or perhaps a component of the Ingenious Armillary Sphere in (5) above.

9. *Ching-fu* ('Shadow Definer'). An ingenious device using the principle of the pin-hole to focus the image of a cross-bar mounted on top of a gnomon, whereby to determine more easily the position of the shadow projected by the apex of the gnomon. The pin-hole was made in the middle of a copper leaf 2 inches wide and 4 inches long, set on an horizontal graduated runner facing the horizontal cross-bar on top of the gnomon. When the cross-bar was seen through the pin-hole to bisect the sun's disc, the position of the device measured the shadow of the cross-bar on the horizontal graduated scale.
10. *K'uei-chi* ('Observing Table'). The text of Kuo Shou-ch'ing's biography in the *Kuo-ch'ao wen-lei* compiled by Su T'ien-chüeh (1294-1352) merely says, 'Although the moon is bright, it is difficult to measure its shadow, (and hence) the *k'uei-chi* was made.' Needham suggests that this was apparently an adaptation of the gnomon and shadow-definer for lunar observations.
11. *Jih-yüeh-shih i* ('Instrument for Observing Solar and Lunar Eclipses'). Our sources give no explanation, but according to Needham this was perhaps the adjustable dioptra described by Arabic writers for determining disc diameters in partial eclipses.
12. *Hsing-kuei* ('Star Dial'). In the absence of factual information, Needham speculates that this could have been the 'nocturnal,' a device for telling the time at night by the fixed stars, consisting of a graduated disc or dial with a hole in the centre and a long projecting arm that could be rotated.
13. *Ting-shih i* ('Time-Determining Instrument'). Little is known about this instrument. Needham suggests that it could either be the 'diurnal circle' of the equatorial torquetum in (1) above, or another name for the Star Dial in (12). Po Shu-ren is inclined to think that (12) and (13) refer to one and the same instrument and that the *Hsing-kuei ting-shi i* was probably a modified form of the *al-Uşūrlāb*.
14. *Cheng-fang an* ('Direction-Determining Table'). This was the azimuthal circle, probably used as a component of the equatorial torquetum in (1) above.
15. *Wan-piao hsüan* ('Suspended Bob-Indicator'). Probably the hanging plumb-line used for checking the trueness of instruments such as the gnomons. Po Shu-ren suggests that (15) should be read as *Wan-piao*, probably a portable type of spherical sun-dial.

16. *Cheng-i* ('Rectifying Instrument'). No description of this instrument is given. If the Suspended Bob-Indicator in (15) above served as a plumb-line, the Rectifying Instrument might have been a water-level, although nothing substantiates this inference. Po Shu-ren thinks that (16) should read *Hsüan cheng-i* ('Suspended Rectifying Instrument') with a suspended plumb-line.
17. *Tso-cheng i* ('Rectifying Instrument on a Stand'). In the absence of any information about it, this instrument may be taken to have been the same as (16) above, mounted on a stand. Po Shu-ren suggests that this was probably an improved version of the water-level.

After the fall of the Yüan, the instruments constructed by Kuo Shou-ching and preserved at the observatory in Ta-tu were removed to Nanking. Between 1437 and 1442, Huang-fu Chung-ho made bronze replicas of all of them. They greatly impressed the Jesuit astronomer Johann Adam Schall von Bell (in Chinese T'ang Jo-wang, 1591-1666), then in the service of the Manchu court. In 1715, however, another Jesuit, Bernard Kilian Stumpf (Chi Li-an 1655-1720), then director of the Astronomical Bureau at Nanking, melted down Kuo's original bronze instruments and also most of Huang-fu's replicas, using the recycled metal to cast new instruments. What remains today are only the Ming replicas of Kuo's equatorial armillary sphere, the equatorial torquetum and the celestial globe.

In February 1278, to expedite the compilation of the new calendar, Qubilai made the calendrical bureau into a full Department of Calendrical Affairs of which Hsü Heng, then senior academician of the Chi-hsien Court, was appointed titular supervisor. The actual administration was entrusted to Wang Hsün, who became director with Kuo Shou-ching as co-director. Wang and Kuo suggested jointly to Qubilai that a new observatory be built at Ta-tu and that it be provided with its own set of bronze astronomical instruments including a gnomon 12 m high to project a long, clear shadow. They further recommended setting up observation stations at five different localities. Qubilai adopted their proposal: he appointed Tuan Chen, vice-minister of Works, to supervise the building of observatories, and later made Aniga, an official of Nepalese origin, responsible for the production of the astronomical instruments. In spring of 1279, a suitable site was chosen close by the eastern adobe wall, outside Chien-kuo men in modern Peking and north of the P'ao-tzu River. Work on the new observatory, exceeding in scale that of Marāgha built in 1259 by Nāṣir al-Dīn Tūsī (1201-74), was then begun. When completed, the complex was provided with offices for the staff not only of the observatory itself but also of the Directorate of Astronomy, with suitable rooms and platforms for housing the various astronomical

instruments, not to mention library and archives, meeting rooms, and even a printing press for printing calendars and almanacs.

All the astronomical instruments were completed and tested that same year. Kuo Shou-ching presented them and explained their functions to Qubilai, who is said to have followed the demonstration untiringly from morning to afternoon. Kuo seized this opportunity to address the emperor on the desirability of two urgent initiatives. The first was to seek experts in calendar science throughout the country to assist in the current task, and the second was to despatch parties of observers to measure the length of gnomon shadows cast by the sun along a transect from the capital through Ho-nan to Nan-hai in south China. Kuo explained that it was essential that shadow-measurements should be carried out over a wider set of latitudes in order to achieve greater accuracy than that obtained by I-hsing and Nan-kung Yüeh during the T'ang dynasty. Qubilai accepted this and appointed fourteen observing officers, who were sent to Koryŏ (Korea) in the east, to T'ien-ch'ih (Yunnan) in the west, to the country of the T'ieh-le (Töles) people in the Irtysh-Yenisei region in the north, and to Chu-ai (in Hainan Island) in the south. Field observers operated at twenty-seven stations, and readings made during the summer solstice at some of the stations were recorded as follows:

Station	North Polar Distance	Observations at Summer Solstice		
		Shadow Length and Direction	Length of day	Length of night
Nan-hai (Kuang-chou, Kwangtung)	15 <i>tu</i> ¹ (14.8°)	1.16 ft S. of gnomon	54 <i>k'o</i>	46 <i>k'o</i> ²
Heng-yüeh (N. of Heng- yang, Hunan)	25 <i>tu</i> (24.6°)	nil; sun directly overhead	56 <i>k'o</i>	44 <i>k'o</i>
Yüeh-t'ai (Hsiang-fu, Honan)	35 <i>tu</i> (34.5°)	1.48 ft N. of gnomon	60 <i>k'o</i>	40 <i>k'o</i>
Ta-tu (Peking)	40 1/12 <i>tu</i> (39.7°)	10.236 ft N. of gnomon ³	62 <i>k'o</i>	38 <i>k'o</i>
Ho-lin (Qara Qorum, Mongolia)	45 <i>tu</i> (44.4°)	3.24 ft N. of gnomon	64 <i>k'o</i>	36 <i>k'o</i>

1. $365.25 \text{ tu} = 360^\circ$.

2. $1 \text{ k'o} = 14.4 \text{ mins} = 1/100\text{th of } 24 \text{ hrs day}$.

3. Apparently a 40-ft-high gnomon was used in the capital instead of the usual 8-ft-high gnomon.

T'ieh-le (Töles)	55 <i>tu</i> (54.2°)	5.10 ft N. of gnomon	70 <i>k'o</i>	30 <i>k'o</i>
Pei-hai (Lake Baikal)	65 <i>tu</i> (64.1°)	6.78 ft N. of gnomon	82 <i>k'o</i>	18 <i>k'o</i>

North Polar distances were also recorded for the following localities:

Shang-tu	43 1/4	<i>tu</i> ¹	(42.6°)
Pei-ching	42 1/12	<i>tu</i>	(42.5°)
I-tu	37 1/4	<i>tu</i>	(36.7°)
Teng-chou	38 1/4	<i>tu</i>	(37.7°)
Kao-li (Korea)	38 1/4	<i>tu</i>	(37.7°)
Hsi-ching	40 1/4	<i>tu</i>	(39.7°)
T'ai-yüan	38 1/4	<i>tu</i>	(37.7°)
An-hsi fu	34 7/8	<i>tu</i>	(34.4°)
Hsing-yüan	33 7/8	<i>tu</i>	(33.4°)
Ch'eng-tu	31 5/8	<i>tu</i>	(31.2°)
Hsi-liang chou	40 1/12	<i>tu</i>	(39.5°)
Tung-p'ing	35 3/4	<i>tu</i>	(35.2°)
Ta-ming	36	<i>tu</i>	(35.5°)
Nan-ching	34 5/6	<i>tu</i>	(34.3°)
Yang-ch'eng	34 2/3	<i>tu</i>	(34.2°)
Yang-chou	33	<i>tu</i>	(32.5°)
E-chou	31 1/2	<i>tu</i>	(31.1°)
Chi-chou	26 1/2	<i>tu</i>	(26.1°)
Lei-chou	19 3/4	<i>tu</i>	(19.5°)
Ch'iung-chou	20 3/4	<i>tu</i>	(26.1°)

That same year, the newly established department also recruited an eminent mathematician, Yang Kung-i (1225-94), who was an expert in calendar science. He made a contribution to the compilation of the new calendar no less significant than that of Kuo Shou-ching and his associates.

The *Shou-shih li*, or Calendar for Fixing the Seasons, was finally completed in spring 1280 and formally presented to the court. With great delight, Qubilai promulgated the calendar at the end of the year and ordered its formal adoption in the first month of the following year. It remained in use until its replacement by the *Ta-i'ung li* towards the end of the Ming dynasty. Kuo Shou-ching and all those engaged in the compilation project were richly rewarded according to their rank. In his memorial to Qubilai accompanying the presentation of the *Shou-shih li* submitted jointly with Yang Kung-i and others, Kuo reported that the author had checked and revised seven old parameters and adopted five new ones. The seven old parameters were:

1. The sub-divisions of the *tu* are after Yabuuti as described in Ho PY66:61-62.

1. *Determination of the exact time of winter solstice for the year 1280*

By taking daily observations of the length of the gnomon's meridian shadow beginning from the 1276 *li-tung* (fortnightly period at mid-point between the autumn equinox and the winter solstice) onward, the winter solstice in 1277 was calculated to occur at 8.5 *k'o* after midnight on the *wu-hsü* day (9/12/1277 at 02.02,2 hours); the summer solstice in the same year, at 70 *k'o* after midnight on the *keng-tzu* day (14/6/1277 at 16.48 hours); the winter solstice in 1278, at 33 *k'o* after midnight on the *kuei-mao* day (9/12/1278 at 07.55,2 hours); the winter solstice in 1279, at 57.5 *k'o* after midnight on the *wu-shen* day (9/12/1279 at 12.08 hours); and the winter solstice 1280, at 81.5 *k'o* after midnight on the *kuei-ch'ou* day (8/12/1280 at 14.33,6 hours). These values differed from those in the original *Ta-ming li* by 18 *k'o* (4 hours 19.2 mins.).

2. *Tropical year*

By taking the estimated time of the winter solstice in six different years recorded over a period of 810 years since the promulgation of the *Ta-ming li*, a value of 365.2425 days was determined for the tropical year.

3. *Positions of the sun*

From the lunar eclipse on the *kuei-yu* day of the 4th lunar month of the *ting-ch'ou* year (8 May 1277), the position of the sun at the winter solstice was calculated to be 10 *tu* (9.8°) from the determinant star of the Chi lunar mansion (γ Sagittarius) on the equator and just over 9 *tu* (8.9°) from the same determinant star on the ecliptic.

4. *Lunar motion*

The daily motion of the moon was recorded from the *ting-ch'ou* year (1277); phases of acceleration and deceleration were noted and the eclipses checked. The *Ta-ming li* was found to be out by 30 *k'o* (7 hours 12 mins).

5. *Nodes*

Nodes were calculated from daily observations of the moon starting from the fifth lunar month of the *ting-ch'ou* year (1277): the times at which the moon's path crossed the ecliptic did not differ much from those predicted by the *Ta-ming li*.

6. Distances from the determinant-stars of the 28 lunar mansions

In the new instruments used for measuring distances from the determinant-stars, each *tu* was divided into 36 *fen*, and the terms *t'ai*, *pan* and *shao* were used to denote 27 *fen*, 18 *fen* and 9 *fen*, respectively.

7. Time of sunrise and sunset

Kuo determined the times of sunrise and sunset on the summer and winter solstices at the Mongol capital of Ta-tu.

The following five new methods were introduced for that purpose by Kuo and his associates:

- i. The method of finite difference was employed to express the irregularities of the sun's angular motion. Because of the elliptical orbit of the earth round the sun at a focus and the non-uniformity of the earth's orbital velocity, the apparent angular motion of the sun is also not uniform. There is a difference between the daily angular motion on any one occasion and the average daily angular motion which is one *tu* or 360/365.25 degree. Previously, two arbitrary constants, known as the *p'ing-ch'a* and the *t'ing-ch'a* were used to represent the differences between the observed values and the average value of the sun's daily angular motion, and these two constants resulted in the solution of quadratic equations. The 7th century astronomer Liu Ch'o was the first to use this method. Kuo Shou-ching, however, introduced a third arbitrary constant, called the *li-ch'a*, resulting in the extraction of the cubic root. If x represents the interval between winter solstice and any given occasion, the difference between observed angular velocity and average angular velocity of the sun can be expressed as a function of a power series of x , i.e. $F(x) = Ax - Bx^2 - Cx^3$, where A , B , and C are the arbitrary constants *p'ing-ch'a*, *ting-ch'a* and *li-ch'a*, respectively. The average daily angular velocity over a period x is given by $F(x)/x = A - Bx - Cx^2$. Kuo observed that the sun moved through 90 degrees (i.e. $365.24/4 = 91.31$ *tu*) in 88.92 days from the last winter solstice. He divided these 88.92 days into six intervals ℓ , each more or less equal to a fortnight, and observed the angular difference at the end of each interval. The results are shown in the following table:

	x No. of days from winter solstice	$F(x)$ Angular diff- erence in 1/10,000 tu p.d.)	$F(x)/x$ Average daily angular differ- ence (in 1/10,000 tu)	Δ^1 First differ- ence	Δ^2 Second differ- ence	Δ^3 Third differ- ence
Initial ($\ell=0$)	0	0	[513.32]*			
1st interval (ℓ)	14.82	7058.0250	476.25	[-37.07]		
2nd interval (2ℓ)	29.64	12976.3920	437.80	-38.45	[-1.38]	[0]
3rd interval (3ℓ)	44.46	17693.7462	397.97	-39.83	-1.38	0
4th interval (4ℓ)	59.28	21148.7328	356.76	-41.21	-1.38	0
5th interval (5ℓ)	74.10	23279.9970	314.17	-42.59	-1.38	0
6th interval (6ℓ)	88.92	24026.1840	270.20	-43.97		

* [] uses data outside the range of columns x and $F(x)$.

Supposing one wished to find the average daily angular difference x days from the last winter solstice. After converting the number of days into the number of periods ℓ , i.e. $\ell = x/14.82$, we have the expression

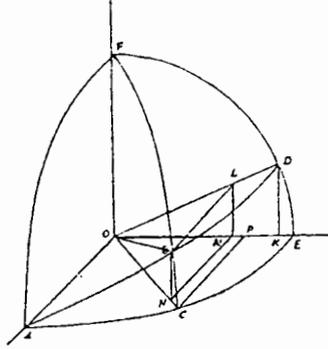
$$\begin{aligned} F(x)/x &= 513.32 + (x/14.82)(-37.07) + 1/2[(x/14.82) - 1](-1.38) \\ &= 513.32 - 2.46x - 0.0031x^2 \end{aligned}$$

$$\text{Hence } F(x) = 513.32x - 2.46x^2 - 0.0031x^3$$

From this equation the angular difference of the sun on any particular day of the year can be calculated.

- ii. The method of finite difference was also applied to calculate the irregularities of the moon's angular motion. This was more accurate than any method used previously in China.
- iii. Before Kuo Shou-ching's time, conversion from ecliptical to equatorial readings were made directly on the armillary sphere and not by calculation. In his 'Simplified Instrument' Kuo and his team had already eliminated the ecliptical component. At the same time he had also used spherical trigonometry to determine (a) the tu - $l\u{u}$, tu on the equator converted from tu on the ecliptic; (b) the chi - $ch'a$, the value of chords for given ecliptic area, and (c) the $ch'a$ - $l\u{u}$, the difference between chords of arcs differing by 1 tu .

In the following figure CE is the equator, BD the ecliptic, BC and DE the two meridian arcs, O the centre of the celestial sphere and D the summer solstice point. $BLMN$ was the rectangle formed by Kuo for his calculations. Let $\angle a$ be the latitude $\angle BOC$; $\angle b$ the equatorial longitude $\angle AOC$; $\angle c$ the ecliptical longitude $\angle AOB$; and $\angle \alpha$ the obliquity of the ecliptic $\angle EOD$.



Expressed in modern mathematics, the following relationships would have been arrived at by Kuo:

$$\sin a = \sin c \sin \alpha \quad (1)$$

$$\cos b = \cos c / \sqrt{\sin^2 c \cos^2 \alpha + \cos^2 c} \quad (2)$$

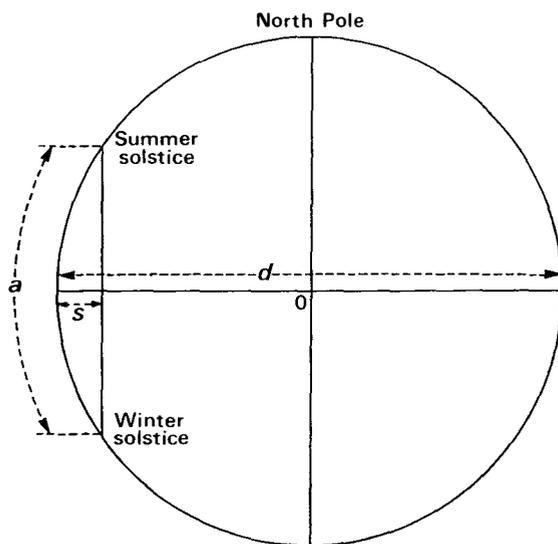
$$\sin b = \sin c \cos \alpha / \sqrt{\sin^2 c \cos^2 \alpha + \cos^2 c} \quad (3)$$

$$\text{Hence } \tan b = \tan c \cos \alpha \quad (4)$$

The whole method was described by him as 'the consideration of right-angled triangles, chords and sagittae, squares and rectangles, all contained, inclined or straight, within circles.' Equations (1) and (4) were used earlier by al-Battānī (ca.858-929). The elaborate calculations employed by Kuo to obtain the same results show that he was not acquainted with the spherical trigonometry used by Muslim astronomers.

- iv. Kuo Shou-ching employed the method of spherical trigonometry to compute accurately the obliquity of the ecliptic. He obtained a value of $23.90 tu$ which is equivalent to $23^\circ 31' 58.5''$. This measurement has an absolute error of only $1' 6.8''$ and was at the time the most accurate ever obtained. In his calculations Kuo used an interesting equation to find the length of an arc a in terms of the diameter d of a circle and the length of the sagitta s in the form:

$$d^2 (a/2)^2 - d^3 s - (d^2 - ad) s^2 + s^4 = 0$$



- v. Kuo Shou-ching determined the positions at which the path of the moon crosses the equator.

Wang Hsün died in 1282, a year after the adoption of the *Shou-shih li*, leaving unfinished the drafts of the methodology used to construct the new calendar. Subsequently, Kuo worked over those notes, from which he produced the following books: *T'ui-pu*, 7 *chüan*; *Li-ch'eng*, 2 *chüan*; *Li-i ni-kao*, 3 *chüan*; *Chuan-shen hsiang-tse*, 2 *chüan*; and *Shang-chung-hsia san-li chu-shih*, 12 *chüan*. Promoted to the post of director of the Department of Calendrical Affairs in early 1286, Kuo Shou-ching presented those works to court together with several other treatises on the calendar and astronomy which he had written during the past few years. They included: *Shih-chien-chu*, 2 *chüan*; *Hsiu-kai yüan-liu*, 1 *chüan*; *I-hsiang fa-shih*, 2 *chüan*; *Erh-kuei-ching k'ao*, 20 *chüan*; *Wu-hsing hsi-hsing k'ao*, 50 *chüan*; *Ku-chin chiao-k'ao*, 1 *chüan*; *Hsin-ts'e erh-shih-pa-she tsa-iso chu-hsing ju-hsiu ch'ü-chi*, 1 *chüan*; *Hsin-ts'e wu-ming chu-hsing*, 1 *chüan*; and *Yüeh-li k'ao*, 1 *chüan*. Unfortunately, only a few of these works survived because the Mongol court, following the Chinese tradition, prohibited the dissemination of astronomical and calendrical treatises to prevent their misuse. Only some of Kuo's writings on the *Shou-shih li*, and instructions for the construction of the astronomical instruments that he and his colleagues had constructed, are quoted in the *Yüan History* (*Yüan-shih*).

There is little information on Kuo Shou-ching's activities until 1291, when he

There is little information on Kuo Shou-ching's activities until 1291, when he was involved in improving transport on the Grand Canal. In that year some officials convinced Qubilai that it would be advantageous to carry grain by water from Ta-tu to Shang-tu. There was one suggestion that the Luan River could be made navigable from Yung-p'ing, and another that a water route could be established on the Lu-kou River. The emperor thereupon sent two expeditions to explore those waterways, and Kuo Shou-ching led the party that surveyed the Lu-kou River. Both expeditions failed to find a route to Shang-tu. However, Kuo later returned to Shang-tu by himself, after which he presented a revised plan to the emperor. This scheme involved the construction of a series of lock-gates on the T'ung-hui section of the Grand Canal between Ta-tu and T'ung-chou. According to Kuo's survey, the rise over this stretch of the country measured some 10 m. We find a description of the plan in Kuo Shou-ching's biography in the *Kuo-ch'ao wen-lei*:

Instead of using the head waters of the I-mou ch'üan spring for the grain shipment to Ta-tu, (Kuo) channelled water from the Po-fu ch'üan source in Pei-shan Mountain towards the west, passing the Weng-shan anchorage to the south, and entering the walled city from the western watergate to run into a reservoir. From there the water flowed eastwards and then to the south to pass the southern watergate into the old canal. A flash-lock (*cha*) was to be established every ten *li*. (Thus from Ta-tu) to T'ung-chou there would be seven flash-lock gates. At a distance of one *li* or more from each flash-lock gate a double lock-gate (*ch'ung-chih tou-men*) would be built, so arranged as to open (*t'i*, lit. 'lift') and shut (*e*) in concert to allow boats through while holding back the water.

Qubilai liked Kuo's idea and ordered that it be carried out immediately.

The same year, while engaged in the Grand Canal project, Kuo Shou-ching recommended to court one Ch'i Lü-ch'ien (1263-1329), an expert in calendar and astronomy, for the post of director of Calendrical Studies. Ch'i was then charged with the responsibility of completing the astronomical instruments that were still under construction. In February 1272, Kuo was appointed director of Waterways. During the second half of the year, construction began on the section of the waterway between T'ung-chou and Ta-tu, consisting of fourteen canals. Twenty-thousand military and civilian labourers were conscripted. Another large canal was constructed joining the nearby rivers for irrigation at Ch'ang-p'ing (Hopei). It is said that Qubilai ordered all the civil servants from the chancellors down to form the workforce which, under Kuo's supervision, was to initiate the project, as a token of the latter's importance. The canals were completed by the autumn of 1293; they greatly expedited communications between T'ung-chou and the capital. Previously, when government food supplies had to be carried by land, many beasts of burden would be lost during the autumn rains. Qubilai rejoiced at the sight of the boats on the canal, which he named T'ung-hui. For his pains, Kuo Shou-ching was awarded 12,500 *min* of paper money, and charged with

administering the new canal, in addition to his other duties. Thereafter, Kuo proposed several other schemes to extend the canal system, without success.

In 1294, shortly before his death, Qubilai raised Kuo Shou-ching to the dignity of senior academician of the Chao-wen Court, and kept him in charge of the Department of Calendrical Affairs. During Qubilai's reign, Kuo made further inventions, such as a 'Seven-Treasures Illuminated Clock' (*ch' i-pao teng-lou*). The clock was provided with bells and drums which sounded on the hour. The qa'an had the instrument set up in the Ta-ming Hall at the palace whenever state functions were held. In addition, Kuo presented to Qubilai models of wheelbarrows that he had devised. These contraptions, named 'Wooden Ox' (*mu-niu*) and 'Gliding Horse' (*liu-ma*), differing somewhat from the ancient designs of Chu-ko Liang (181-234), were quite ingenious.

In 1298, Temür (Ch'eng-tsung, r. 1294-1307), the new qa'an, on being reminded of Kuo Shou-ching's earlier proposals, summoned Kuo to Ta-tu to plan a new canal south of Shang-tu, which would divert waters from T'ieh-fan kan Peak (north of Tushih k'ou in Ch'ih-ch'eng county, Hopei) to the Luan River. Kuo advised that the canal should be at least 50 to 70 *pu* (=77 to 107 m) wide to carry the summer surge. But senior officials wishing to save costs had the width of the canal reduced by one-third against Kuo's recommendation. Duly completed, the canal broke its banks in 1299, causing great loss of life and property, and almost flooding the imperial palace. Greatly disturbed, Temür expressed deep regret for not having listened to Kuo Shou-ching. Also in 1298, Kuo constructed a water-clock-driven armillary sphere. This comprised some twenty-five wooden cog-wheels of various sizes, linked together so as to drive a celestial sphere marked with the positions of the stars. Two rings circled the sphere, marking the paths of the sun and the moon. The sphere turned anticlockwise following the movement of the heavens, while the two rings revolved clockwise to simulate the movements of the heavenly bodies. Furthermore, Kuo attempted to reconstruct Chang Heng's (*ca.* 100 AD) seismoscope (*ti-ung i*), without success.

In 1303, all the civil servants aged seventy *sui* or more were ordered to retire. Yet Temür kept Kuo in service because the latter's expertise and experience were still needed. In fact, all the Han-lin academicians and officials of the Department of Calendrical Affairs and the Directorate of Astronomy were thereafter exempted from mandatory retirement. Despite his official rank, Kuo Shou-ching chose to live henceforward in virtual retirement. His only significant contribution thereafter was the construction of the 'Cabinet Incense-Clock' (*kuei-hsiang lou*) and the 'Movable Door-Screen Incense-Clock' (*p'ing-feng hsiang lou*), presumably two types of combustion clocks. In addition, he also made a small 'Portable Clepsydra' (*hsing lou*),

for Temür's use while performing sacrifices to Heaven (*chiao-miao*), and on his travels.

Kuo Shou-ching died in 1316 at the age of eighty-five, and was buried at a spot about 16 km north-west of his birthplace, Hsin-t'ai. His disciple Ch'i Lü-ch'ien wrote an 'account of conduct' (*hsing-chuang*) in Kuo's memory, and this became the main source of his biography in the *Kuo-ch'ao wen-lei* as well as the *Yüan History*. Little is known about Kuo's posterity; a great-grandson, Kuo Kuei, served as an official in the Directorate of Astronomy during the T'ien-shun era (1457-64) of the Ming emperor Ying-tsung and contributed to the preparation of the *Ta-t'ung li* based on the *Shou-shih li*, thus continuing the calendrical work of his distinguished forebear.

In light of his many ingenious and original contributions, Kuo Shou-ching must be hailed as the most outstanding hydrologist and astronomer of the Yüan dynasty. In the 17th century, Johann Adam Schall von Bell called him 'the Tycho Brahe of China' after having examined some of the astronomical instruments constructed by Kuo. Indeed, while he undoubtedly benefitted from the assistance of several contemporary scientists, Kuo Shou-ching may nevertheless be regarded as the last – but only in the sense of time – of the great traditional astronomers and mathematicians in Chinese history.

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P.Y. Ho

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With the assistance of May Wang



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The signet shows a Mongol archer from the 14th century Chinese encyclopaedia *Shih-lin kuang-chi*.

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